

Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics


Further Pure Mathematics FP1 (6674)

June 2007
6674 Further Pure Mathematics FP1
Mark Scheme

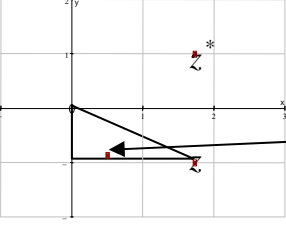
Question number	Scheme	Marks
1.	<p>$1\frac{1}{2}$ and 3 are ‘critical values’, e.g. used in solution, or both seen as asymptotes</p> <p>$(x+1)(x-3) = 2x-3 \Rightarrow x(x-4) = 0$</p> <p>$x = 4, x = 0$ M1: attempt to find at least one other critical value</p> <p>$0 < x < 1\frac{1}{2}, 3 < x < 4$ M1: An inequality using $1\frac{1}{2}$ or 3</p>	<p>B1</p> <p>M1 A1, A1</p> <p>M1 A1, A1 (7)</p> <p>7</p>
	<p>First M mark can be implied by the two correct values, but otherwise a method must be seen. (The method may be graphical, but either $(x =) 4$ or $(x =) 0$ needs to be clearly written or used in this case).</p> <p>Ignore ‘extra values’ which might arise through ‘squaring both sides’ methods.</p> <p>≤ appearing: maximum one A mark penalty (final mark).</p>	

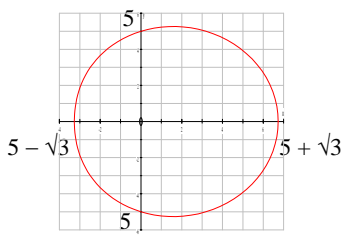
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2.	<p>Integrating factor $e^{\int -\tan x dx} = e^{\ln(\cos x)}$ (or $e^{-\ln(\sec x)}$), $= \cos x$ (or $\frac{1}{\sec x}$)</p> <p>$\left(\cos x \frac{dy}{dx} - y \sin x = 2 \sec^2 x \right)$</p> <p>$y \cos x = \int 2 \sec^2 x dx$ (or equiv.) (Or: $\frac{d}{dx}(y \cos x) = 2 \sec^2 x$)</p> <p>$y \cos x = 2 \tan x (+C)$ (or equiv.)</p> <p>$y = 3$ at $x = 0$: $C = 3$</p> <p>$y = \frac{2 \tan x + 3}{\cos x}$ (Or equiv. in the form $y = f(x)$)</p>	<p>M1, A1</p> <p>M1 A1(ft)</p> <p>A1</p> <p>M1</p> <p>A1 (7)</p> <p>7</p>
	<p>1st M: Also scored for $e^{\int \tan x dx} = e^{-\ln(\cos x)}$ (or $e^{\ln(\sec x)}$), then A0 for $\sec x$.</p> <p>2nd M: Attempt to use their integrating factor (requires one side of the equation 'correct' for their integrating factor).</p> <p>2nd A: The follow-through is allowed <u>only</u> in the case where the integrating factor used is $\sec x$ or $-\sec x$. $\left(y \sec x = \int 2 \sec^4 x dx \right)$</p> <p>3rd M: Using $y = 3$ at $x = 0$ to find a value for C (dependent on an integration attempt, however poor, on the RHS).</p> <p><u>Alternative</u></p> <p>1st M: Multiply through the given equation by $\cos x$.</p> <p>1st A: Achieving $\cos x \frac{dy}{dx} - y \sin x = 2 \sec^2 x$. (Allowing the possibility of integrating by inspection).</p>	

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3.	<p>(a) $(r+1)^3 = r^3 + 3r^2 + 3r + 1$ and $(r-1)^3 = r^3 - 3r^2 + 3r - 1$</p> <p>$(r+1)^3 - (r-1)^3 = 6r^2 + 2$ (*)</p> <p>(b) $r = 1: 2^3 - 0^3 = 6(1^2) + 2$ $r = 2: 3^3 - 1^3 = 6(2^2) + 2$ $\vdots \vdots \vdots \vdots \vdots \vdots \vdots \vdots \vdots \vdots$ $r = n: (n+1)^3 - (n-1)^3 = 6n^2 + 2$ M: Differences: at least first, last and one other.</p> <p>Sum: $(n+1)^3 + n^3 - 1 = 6\sum r^2 + 2n$ M: Attempt to sum at least one side.</p> <p>$(6\sum r^2 = 2n^3 + 3n^2 + n)$</p> <p>$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$ (Intermediate steps are not required) (*)</p> <p>(c) $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2, = \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}(n-1)n(2n-1)$</p> <p>$= \frac{1}{6}n((16n^2 + 12n + 2) - (2n^2 - 3n + 1))$</p> <p>$= \frac{1}{6}n(n+1)(14n+1)$</p>	<p>M1</p> <p>A1cso (2)</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1cso (5)</p> <p>M1, A1</p> <p>M1</p> <p>A1 (4)</p> <p>11</p>
	<p>(b) 1st A: Requires first, last and one other term correct on both LHS and RHS (but condone 'omissions' if following work is convincing).</p> <p>(c) 1st M: Allow also for $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^n r^2$.</p> <p>2nd M: Taking out (at some stage) factor $\frac{1}{6}n$, and multiplying out brackets to reach an expression involving n^2 terms.</p>	

Question number	Scheme	Marks
4.	<p>(a) $f'(x) = 3x^2 + 8$ $3x^2 + 8 = 0 \dots\dots$ or $3x^2 + 8 > 0 \dots\dots$ Correct derivative and, e.g., 'no turning points' or 'increasing function'.</p>  <p>Simple sketch, (increasing, crossing positive x-axis) (or, if the M1 A1 has been scored, a <u>reason</u> such as 'crosses x-axis only once').</p> <p>(b) Calculate $f(1)$ and $f(2)$ (<u>Values</u> must be seen) $f(1) = -10$, $f(2) = 5$, Sign change, \therefore Root</p> <p>(c) $x_1 = 2 - \frac{f(2)}{f'(2)}$, $= 2 - \frac{5}{20}$ (= 1.75)</p> <p>$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$, $\left(= 1.75 - \frac{0.359375}{17.1875} \right) = 1.729$ (ONLY) (α)</p> <p>(d) Calculate $f(\alpha - 0.0005)$ and $f(\alpha + 0.0005)$ (or a 'tighter' interval that gives a sign change). $f(1.7285) = -0.0077\dots$ and $f(1.7295) = 0.0092\dots$, \therefore Accurate to 3 d.p.</p>	<p>M1 A1 B1 (3)</p> <p>M1 A1 (2)</p> <p>M1, A1</p> <p>M1, A1 (4)</p> <p>M1</p> <p>A1 (2)</p> <p>11</p>
	<p>(a) M: Differentiate and consider sign of $f'(x)$, or equate $f'(x)$ to zero. <u>Alternative:</u> M1: Attempt to rearrange as $x^3 - 19 = -8x$ or $x^3 = 19 - 8x$ (condone sign slips), and to sketch a cubic graph and a straight line graph. A1: Correct graphs (shape correct and intercepts 'in the right place'). B1: Comment such as "one intersection, therefore one root").</p> <p>(c) 1st A1 can be implied by an answer of 1.729, provided N.R. has been used. <u>Answer only:</u> No marks. The Newton-Raphson method must be seen.</p> <p>(d) For A1, correct <u>values</u> of $f(1.7285)$ and $f(1.7295)$ must be seen, together with a conclusion. If only 1 s.f. is given in the values, allow rounded (e.g. -0.008) <u>or</u> truncated (e.g. -0.007) values.</p>	

Question number	Scheme	Marks
5.	<p>C.F. $m^2 + 3m + 2 = 0$ $m = -1$ and $m = -2$</p> <p>$y = Ae^{-x} + Be^{-2x}$</p> <p>P.I. $y = cx^2 + dx + e$</p> <p>$\frac{dy}{dx} = 2cx + d, \frac{d^2y}{dx^2} = 2c$ $2c + 3(2cx + d) + 2(cx^2 + dx + e) \equiv 2x^2 + 6x$</p> <p>$2c = 2$ $c = 1$ (One correct value)</p> <p>$6c + 2d = 6$ $d = 0$</p> <p>$2c + 3d + 2e = 0$ $e = -1$ (Other two correct values)</p> <p>General soln: $y = Ae^{-x} + Be^{-2x} + x^2 - 1$ (Their C.F. + their P.I.)</p> <p>$x = 0, y = 1: 1 = A + B - 1$ ($A + B = 2$)</p> <p>$\frac{dy}{dx} = -Ae^{-x} - 2Be^{-2x} + 2x, x = 0, \frac{dy}{dx} = 1: 1 = -A - 2B$</p> <p>Solving simultaneously: $A = 5$ and $B = -3$</p> <p>Solution: $y = 5e^{-x} - 3e^{-2x} + x^2 - 1$</p>	<p>M1</p> <p>A1 (2)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1ft (5)</p> <p>M1</p> <p>M1</p> <p>M1 A1</p> <p>A1 (5)</p> <p>12</p>
	<p>1st M: Attempt to solve auxiliary equation.</p> <p>2nd M: Substitute their $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the D.E. to form an identity in x with unknown constants.</p> <p>3rd M: Using $y = 1$ at $x = 0$ in their general solution to find an equation in A and B.</p> <p>4th M: Differentiating their general solution (condone 'slips', but the <u>powers</u> of each term must be correct) and using $\frac{dy}{dx} = 1$ at $x = 0$ to find an equation in A and B.</p> <p>5th M: Solving simultaneous equations to find both a value of A and a value of B.</p>	

Question number	Scheme	Marks
6.	<p>(a) $z^* = \sqrt{3} + i$ $\frac{z}{z^*} = \frac{(\sqrt{3}-i)(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)} = \frac{3-2\sqrt{3}i-1}{3+1} = \frac{1-\sqrt{3}i}{2}$ (*)</p> <p>(b) $\left \frac{z}{z^*} \right = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\pm\sqrt{3}}{2}\right)^2} = 1$ [Or: $\left \frac{z}{z^*} \right = \frac{ z }{ z^* } = \frac{\sqrt{3+1}}{\sqrt{3+1}} = 1$]</p> <p>(c) $\arg(w) = \arctan\left(\pm \frac{\text{imag}(w)}{\text{real}(w)}\right)$ or $\arg(w) = \arctan\left(\pm \frac{\text{real}(w)}{\text{imag}(w)}\right)$, where w is z or z^* or $\frac{z}{z^*}$</p> <p>$\arg\left(\frac{z}{z^*}\right) = \arctan\left(\frac{-\sqrt{3}/2}{1/2}\right) = -\frac{\pi}{3}$</p> <p>$\arctan\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ and $\arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ (Ignore interchanged z and z^*)</p> <p>$\arg z - \arg z^* = -\frac{\pi}{6} - \frac{\pi}{6} = -\frac{\pi}{3} = \arg\left(\frac{z}{z^*}\right)$</p> <p>(d)  z and z^* (Correct quadrants, approx. symmetrical)</p> <p>$\frac{z}{z^*}$ (Strictly <u>inside</u> the triangle shown here)</p> <p>(e) $(x - (\sqrt{3} - i))(x - (\sqrt{3} + i))$ Or: Use sum of roots $\left(= \frac{-b}{a} \right)$ and product of roots $\left(= \frac{c}{a} \right)$. $x^2 - 2\sqrt{3}x + 4$</p>	<p>B1</p> <p>M1, A1cso (3)</p> <p>M1, A1 (2)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1 (4)</p> <p>B1</p> <p>B1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>13</p>
	<p>(a) M: Multiplying both numerator and denominator by $\sqrt{3} - i$, and multiplying out brackets with <u>some</u> use of $i^2 = -1$.</p> <p>(b) Answer 1 with no working scores both marks.</p> <p>(c) Allow work in degrees: -60°, -30° and 30° Allow arg between 0 and 2π: $\frac{5\pi}{3}$, $\frac{11\pi}{6}$ and $\frac{\pi}{6}$ (or 300°, 330° and 30°). Decimals: Allow marks for awrt -1.05 (A1), -0.524 and 0.524 (A1), but then A0 for final mark. (Similarly for 5.24 (A1), 5.76 and 0.524 (A1)).</p> <p>(d) Condone wrong labelling (or lack of labelling), if the intention is clear.</p>	

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7.	<p>(a) </p> <p>Shape (closed curve, approx. symmetrical about the initial line, in all 'quadrants' and 'centred' to the right of the pole/origin).</p> <p>Scale (at least one correct 'intercept' r value... shown on sketch or perhaps seen in a table). (Also allow awrt 3.27 or awrt 6.73).</p> <p>(b) $y = r \sin \theta = 5 \sin \theta + \sqrt{3} \sin \theta \cos \theta$ $\frac{dy}{d\theta} = 5 \cos \theta - \sqrt{3} \sin^2 \theta + \sqrt{3} \cos^2 \theta \quad (= 5 \cos \theta + \sqrt{3} \cos 2\theta)$ $5 \cos \theta - \sqrt{3}(1 - \cos^2 \theta) + \sqrt{3} \cos^2 \theta = 0$ $2\sqrt{3} \cos^2 \theta + 5 \cos \theta - \sqrt{3} = 0$ $(2\sqrt{3} \cos \theta - 1)(\cos \theta + \sqrt{3}) = 0 \quad \cos \theta = \dots (0.288\dots)$ $\theta = 1.28$ and 5.01 (awrt) (Allow ± 1.28 awrt) (Also allow $\pm \arccos \frac{1}{2\sqrt{3}}$) $r = 5 + \sqrt{3} \left(\frac{1}{2\sqrt{3}} \right) = \frac{11}{2}$ (Allow awrt 5.50)</p> <p>(c) $r^2 = 25 + 10\sqrt{3} \cos \theta + 3 \cos^2 \theta$ $\int 25 + 10\sqrt{3} \cos \theta + 3 \cos^2 \theta d\theta = \frac{53\theta}{2} + 10\sqrt{3} \sin \theta + 3 \left(\frac{\sin 2\theta}{4} \right)$ (ft for integration of $(a + b \cos \theta)$ and $c \cos 2\theta$ respectively) $\frac{1}{2} \left[25\theta + 10\sqrt{3} \sin \theta + \frac{3 \sin 2\theta}{4} + \frac{3\theta}{2} \right]_0^{2\pi} = \dots$ $= \frac{1}{2} (50\pi + 3\pi) = \frac{53\pi}{2}$ or equiv. in terms of π.</p>	<p>B1</p> <p>B1 (2)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (6)</p> <p>B1</p> <p>M1 <u>A1ft</u> <u>A1ft</u></p> <p>M1</p> <p>A1 (6)</p> <p>14</p>
	<p>(b) 2nd M: Forming a quadratic in $\cos \theta$. 3rd M: Solving a 3 term quadratic to find a value of $\cos \theta$ (even if called θ). <u>Special case:</u> Working with $r \cos \theta$ instead of $r \sin \theta$: 1st M1 for $r \cos \theta = 5 \cos \theta + \sqrt{3} \cos^2 \theta$ 1st A1 for derivative $-5 \sin \theta - 2\sqrt{3} \sin \theta \cos \theta$, then no further marks.</p> <p>(c) 1st M: Attempt to integrate at least one term. 2nd M: Requires use of the $\frac{1}{2}$, correct limits (which could be 0 to 2π, or $-\pi$ to π, or 'double' 0 to π), and subtraction (which could be implied).</p>	